 LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - MATHEMATICS

THIRD SEMESTER – APRIL 2012

# MT 3811 - COMPLEX ANALYSIS

Date : 24-04-2012 Dept. No. Max. : 100 Marks

Time : 1:00 - 4:00

Answer all the questions.

1. a) Prove that if using Leibniz’s rule

OR

b) Let be a non-constant polynomial. Prove that there is a complex number such that. (5)

c) Let be an analytic function. Prove that for where Hence prove that if f is analytic in an open disk then prove that for where.

OR

d) State and prove homotopic version of Cauchy’s theorem. (15)

2. a) State and prove Morera’s theorem.

OR

b) Prove that a differentiable function on is convex if and only if is

increasing. (5)

c) State and prove the Arzela-Ascoli theorem.

OR

d) State and prove the Riemann mapping theorem.

(15)

3. a) Show that in the usual notation.

OR

b) If and then prove that .

(5)

c) (i) Let be a compact metric space and let be a sequence of continuous functions from *X* into such that converges absolutely and uniformly for *x* in *X*. Then prove that the product converges absolutely and uniformly for *x* in *X*. Also prove that there is an integer such that if and only if for some *n*, .

(ii) State and prove Weierstrass factorization theorem. (7+8)

OR

d) Let , then prove that converges absolutely if and only if converges absolutely.

e) State and prove Bohr-Mollerup theorem. (7+8)

4. a) State and prove Jensen’s formula.

OR

b) If is a metric space, then prove that is also a metric on (5)

c) State and prove Rung’s theorem. (15)

OR

d) State and prove Hadamard’s factorization theorem. (15)

5. a) Prove that any two bases of a same module are connected by a unimodular transformation.

OR

b) Prove that an elliptic function without poles is a constant.

(5)

c) (i) Prove that the zeros and poles of an elliptic function satisfy .

(ii) Derive Legendre’s relation (7+8)

OR

d) (i) State and prove the addition theorem for the Weierstrassfunction.

(ii) Show that

(8+7)

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*